6.5 Scalar Product

573. Scalar Product of Vectors $\vec{u}$ and $\vec{v}$

$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$,

where θ is the angle between vectors $\vec{u}$ and $\vec{v}$.

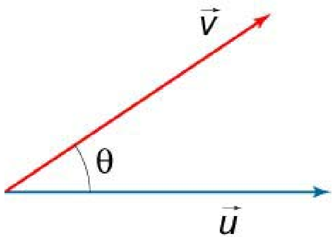

Figure 82.

574. Scalar Product in Coordinate Form

If $\vec{u} = (X_1, Y_1, Z_1)$, $\vec{v} = (X_2, Y_2, Z_2)$, then

$\vec{u} \cdot \vec{v} = X_1X_2 + Y_1Y_2 + Z_1Z_2$.

575. Angle Between Two Vectors

If $\vec{u} = (X_1, Y_1, Z_1)$, $\vec{v} = (X_2, Y_2, Z_2)$, then

$$\cos\theta = \frac{X_1X_2 + Y_1Y_2 + Z_1Z_2}{\sqrt{X_1^2 + Y_1^2 + Z_1^2}\sqrt{X_2^2 + Y_2^2 + Z_2^2}}.$$

576. Commutative Property

$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

577. Associative Property

$(\lambda\vec{u}) \cdot (\mu\vec{v}) = \lambda\mu\vec{u} \cdot \vec{v}$

578. Distributive Property

$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

579. $\vec{u} \cdot \vec{v} = 0$ if $\vec{u}, \vec{v}$ are orthogonal $(\theta = \frac{\pi}{2})$.

580. $\vec{u} \cdot \vec{v} > 0$ if $0 < \theta < \frac{\pi}{2}$.

581. $\vec{u} \cdot \vec{v} < 0$ if $\frac{\pi}{2} < \theta < \pi$.

582. $\vec{u} \cdot \vec{v} \leq |\vec{u}| \cdot |\vec{v}|$

583. $\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}|$ if $\vec{u}, \vec{v}$ are parallel $(\theta = 0)$.

584. If $\vec{u} = (X_1, Y_1, Z_1)$, then
$\vec{u} \cdot \vec{u} = \vec{u}^2 = |\vec{u}|^2 = X_1^2 + Y_1^2 + Z_1^2$.

585. $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$

586. $\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$